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Non-supersymmetric SO(3)-invariant deformations of $\mathcal{N} = 1^{\star}$ vacua and their dual string theory description

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Abstract: We study the SO(3)-invariant relevant deformations of $\mathcal{N} = 4$ SU(N) gauge theory using the methods of Polchinski and Strassler. We present the region of parameter space where the non-supersymmetric vacuum is still described by stable "dielectric" five branes within the supergravity approximation.

Keywords: D-branes, AdS-CFT Correspondance, Supersymmetry Breaking.

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Contents

1. Introduction

Via the AdS/CFT correspondence[1] one can study relevant deformations of four dimensional $\mathcal{N}=4$ SU(N) gauge theory at strong 't Hooft coupling. Using this technique we have obtained new insights about RG flows [2]–[17], IR fixed points and the c-theorem [18, 3, 19, 5, 20, 21] and possible chiral symmetry breaking [22]. But we have also learned that, unless the perturbations are fine tuned to end up at some stable IR fixed point (which up to now always seems to require some unbroken supersymmetry), the spacetime geometry usually develops a naked singularity deep into the bulk.[2] Close to it, which corresponds to the field theory low energy physics, supergravity ceases to be valid and stringy corrections become important.

In the case of $\mathcal{N}=1$ supersymmetric mass deformations, the so called $\mathcal{N}=1^\star$ field theory, Polchinski and Strassler [24] made a precise proposal for the string theory resolution of the singularity: it is replaced by a five brane configuration.[3] In principle, for any $\mathcal{N}=1^\star$ field theory vacuum, there is a dual description in terms of a type-IIB background involving several "dielectric" five branes with D3 charges. The brane configurations create some type-IIB string theory boundary conditions,

[1] See for instance [1] for a review.
[2] See [23] for a discussion on these singularities.
[3] See [25]–[28] for some recent extensions of [24].

and we know how to read the dictionary between particular boundary conditions and field theory bare Lagrangians. To obtain $\mathcal{N} = 1^\star$ Lagrangians, we need non-trivial boundary conditions for the magnetic three-form. Their sources are five branes with non-zero electric dipoles.

Once this set up was established, Polchinski and Strassler proceeded to find a computable dual string theory description of the $\mathcal{N} = 1^*$ field theory vacua. Their approach was to take the most similar vacuum in the $\mathcal{N} = 4$ Coulomb branch, a continuous distribution of D3 branes with the topology of a two sphere, and compute the linearized perturbed background caused by the introduction of non-vanishing boundary conditions to the three-form (the fermion masses). It results that, in certain regions of parameter space, the linearized solution is enough to have a valid description. In presence of the magnetic three-form, the D3 branes are replaced by several five branes via the Myers effect [29]. Then, the whole problem is reduced to find the five brane configuration that produces the perturbed type-IIB background. A good criteria is to look for those configurations which minimize the five brane effective action in such a background. It was realized in [24] that if the ratio of five and three brane charge densities is very small, the linearized solution becomes a valid approximation around the five brane action minima.[4]

Besides this computational condition, which we refer as the PS approximation, there are the usual constraints for the classical supergravity regime: small curvature in Planck and string units. Now, we have to be more careful, because the brane configuration produces non-constant curvature and dilaton backgrounds. In fact, close to the five branes, we have to interpolate from the three brane distribution metric to the near-shell five brane metric with D3 charge. For instance, for the case of a D5 brane, the PS approximation corresponds to

$$1 \ll \frac{N}{g} \ll N^2 \tag{1.1}$$

and for a NS5 brane, we have the S-dual condition

$$1 \ll gN \ll N^2 \,. \tag{1.2}$$

As it was already noticed in [24], the regime of validity of their approximation can be extended to certain non-supersymmetric directions in the parameter space. In this paper, we look for those non-supersymmetric directions which preserve SO(3) invariance and analyse the region which is still within the regime of validity of the PS approximation. We will complete the determination of the effective potential started in [24] and study the resulting vacuum structure.

[4] For completeness, we should mention that there is a second order term (in the fermion masses) which should be included in the five brane action. But supersymmetry allows one to obtain it without having to compute it explicitly.

This paper grew out of an attempt [30] to apply the methods of [24] to the non-supersymmetric theory with dynamically-broken chiral symmetry, discussed in [22] (in the notation below, this is the theory with $m = \mu^2 = 0$, and $m' \neq 0$). Unfortunately, the PS approximation proved not to be valid for that theory. Hence the motivation for the present work: to find the precise realm of validity of the PS approximation within the space of non-supersymmetric, SO(3)-invariant extensions of [24].

2. $\mathcal{N} = 0$ SO(3)-invariant five brane potential

2.1 PS approximation

We begin by presenting the relevant effective potential for a five brane probe within the PS approximation:

$$-S = \frac{\mu_5 V}{g}\Big\{|M|^2 \int_{S^2} d^2\xi \frac{\sqrt{\det G_\|}\det G_\perp}{4\pi\alpha'\sqrt{\det F_2}} + \frac{\sqrt{2}}{3}\int_{S^2} \mathrm{Im}\left(\overline{M} T_{mnp} x^m dx^n \wedge dx^p\right) + \\ + \pi\alpha' \int_{S^2} F_2 \left(T_{i\bar{j}\bar{k}} \overline{T}_{\bar{l}jk} z^i \overline{z}^{\bar{l}} + \mu_{mn} x^m x^n\right)\Big\}. \tag{2.1}$$

In this subsection we will explain what are the fields entering into this potential, where it comes from, and when it is a good approximation.

First, we introduce the relevant fields entering in (2.1).[5] Since the fermion mass matrix transform in the $\mathbf{10}^\star$ of SU(4), it can be represented by an imaginary anti-self-dual three form

$$\star_6 T_3 = -i T_3\,, \tag{2.2}$$

where $\star_6$ acts in the six dimensional transverse space, parametrized by x^m ($m = 4,\ldots,9$), with respect to the flat euclidean metric δ_{mn}. Because $\mathbf{10}^\star$ is a complex representation, it is convenient to use the complex coordinates

$$z^i = \frac{x^{i+3} + i x^{i+6}}{\sqrt{2}}\,, \quad i = 1,2,3\,. \tag{2.3}$$

For the SO(3) invariant case, where m is the $\mathcal{N} = 1$ preserving mass of the three chiral superfields, and m' is the gaugino mass (which breaks supersymmetry softly), the anti-self-dual form becomes

$$T_3 = m\epsilon_{ijk} dz^i \wedge d\overline{z}^j \wedge d\overline{z}^k + m' dz^1 \wedge dz^2 \wedge dz^3\,. \tag{2.4}$$

The ten dimensional (string) metric is of the form

$$ds^2 = \frac{1}{\sqrt{Z}} dx_\|^2 + \sqrt{Z} dx^m dx^m\,. \tag{2.5}$$

[5] We follow the conventions of [24] faithfully.

When $r^2 = x^m x^m \to \infty$, we have the $AdS_5 \times S^5$ boundary conditions

$$
\begin{aligned}
&Z \to \frac{4\pi\alpha'^2}{r^4}\,, \\
&e^{\Phi} \to g\,, \qquad C \to \frac{\theta}{2\pi} \qquad \text{(constant dilaton and axion)}\,, \\
&F_5 \to d\chi_4 + \star d\chi_4\,, \qquad \chi_4 = \frac{1}{gZ} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3\,, \\
\star_6 G_3 - iG_3 &\to -i\frac{2\sqrt{2}}{g} Z T_3\,,
\end{aligned}
\tag{2.6}
$$

where $G_3 = F_3 - \tau H_3$ and $\tau = C + ie^{-\Phi}$ is the usual complex field theory coupling; F_3 and H_3 are the RR and NS-NS three-form field strengths, respectively.

We are considering a five brane probe, with D3 brane charge n, in type-IIB background solution of the supergravity equations of motion subject to the above boundary conditions. Due to the particular expression for the non-zero six-form potentials determined by (2.6), the five brane world-volume is taken to be $\mathbb{R}^4 \times S^2$, with the two sphere in the six dimensional transverse space. $G_{||}$ and $G_{\perp}$ correspond to the target spacetime metrics in the directions $\mathbb{R}^4$ and S^2, respectively. The Born-Infeld U(1) field, F_2, has a non-zero flux around the two sphere, giving the D3 brane charge,

$$
\int_{S^2} F_2 = 2\pi n\,. \tag{2.7}
$$

Finally, $M = c\tau + d$ for a (c, d) five brane and V is the (regularized) R^4 coordinate volume. We will discuss on the constant matrix μ_{mn} later.

Next, we explain the origin of the terms appearing in (2.1). The first term corresponds to an expansion of the Born-Infeld square-root term due to the small five brane metric $G_{\perp}$ corrections with respect to the dominant D3 charge given by F_2. Observe that the Z factors cancel explicitly, because of the particular ansatz for the metric (2.5). The second term comes from the linearized (in the fermion masses) six-form potential $B_6 c + C_6 d$. The PS analysis showed that it is independent of the metric factor Z.

To determine the third term would require more work. It comes from the second order solution, in the fermion masses, for the supergravity fields. It should be included, since posteriori analysis on the location of the minimum shows that this term is actually of the same order as the previous two terms in the action. The traceless matrix μ_{mn} represents the $L = 2$ harmonic in the transverse five dimensional compact space. It receives contributions form the dilaton, spacetime metric and five-form field strength. When $\mathcal{N} = 1$ supersymmetry is unbroken, and we take the round ansatz $F_2 = n/2 \sin\theta$ for the SO(3) invariant case, the potential (2.1) should be a perfect square in terms of the complex scalar fields z^i. This requirement determines $\mu_{mn} = 0$. In our case, we lack supersymmetry and we should find some other way to determine μ_{mn}.

Before doing that, let us finish this subsection with a comment on the validity of the potential (2.1). A close analysis shows that the full five brane action has been expanded in the controlling parameter $g|M|^2/n$. The PS approximation consists in going to the region of parameter space where it is small. Our goal is to find out what that region is for the $\mathcal{N}=0$ SO(3)-invariant case and to analyze the corresponding vacuum structure therein.

2.2 $\mathcal{N}=0^*$ SO(3)-invariant deformations

There are three complex parameters parametrizing the SO(3)-invariant relevant deformations. The branching rules of $\mathrm{SU}(4) \to \mathrm{SU}(3) \times \mathrm{U}(1)_{\mathrm{R}}$ are

$$\begin{aligned}
\mathbf{4} &\to \mathbf{3}_{-1/3} + \mathbf{1}_1 \,, \\
\mathbf{6} &\to \mathbf{3}_{2/3} + \mathbf{3}^\star{}_{-2/3} \,, \\
\mathbf{10}^\star &\to \mathbf{6}^\star{}_{2/3} + \mathbf{3}^\star{}_{-2/3} + \mathbf{1}_{-2} \,, \\
\mathbf{20}' &\to \mathbf{8}_0 + \mathbf{6}_{4/3} + \mathbf{6}^\star{}_{-4/3} \,.
\end{aligned} \tag{2.8}$$

The abelian factor $\mathrm{U}(1)_{\mathrm{R}}$ is the R-symmetry of the $\mathcal{N}=1^\star$ field theory. The gluino, which is the SU(3) singlet in the $\mathbf{4}$, has R-charge one. The complex singlet $\mathbf{1}_{-2}$ in $\mathbf{10}^\star$ corresponds to m', the supersymmetry breaking gluino mass. The three $\mathcal{N}=1$ chiral superfields are the $\mathbf{3}_{2/3}$, with the $\mathbf{3}_{-1/3}$ from the $\mathbf{4}$ being its fermion components. The $\mathcal{N}=1$ preserving mass matrix corresponds to the $\mathbf{6}^\star{}_{2/3}$. When its three eigenvalues are equal to m, there is an unbroken SO(3), which is the real embedded subgroup of SU(3), up to a global complex phase (the phase of m). The $\mathbf{20}'$ represents a traceless mass matrix for the six scalars and can also been understood as the $L=2$ harmonic μ_{mn} in (2.1).

As it is observed in [5, 24], the $L=2$ harmonic only enters in the homogeneous part of the supergravity equations of motion. The inhomogeneous term is determined by the anti-self-dual three form T_3, which in our case includes the parameters m and m'. In the case of the $\mathcal{N}=1$ SO(3) boundary conditions, the particular solution gives $\mu_{mn}=0$. But there are two additional SO(3) singlets in the $\mathbf{20}'$, which break supersymmetry when they are turned on. This change in the boundary conditions can be effectively parametrized by a single complex parameter μ^2 (dimension mass square), which determines the $L=2$ harmonic in (2.1) to be

$$\mu_{mn} x^m x^n = \mathrm{Re}[\mu^2 z^i z^i] \,. \tag{2.9}$$

Observe that $\mathrm{U}(1)_{\mathrm{R}}$ selection rules would allow a possible additional $\mathrm{Re}[mm' z^i z^i]$ term. But SO(4) symmetry when $\mu^2=0$ and $m=m'$ (which enters into the region of PS approximation, see subsection 3.3) eliminates this possibility.[6]

[6]We would like to thank M. Strassler for a discussion on this point.

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2.3 The potential and its critical points

Because of the SO(3) symmetry, we take the same ansatz as in [24]

$$z^i = z e^i \,, \tag{2.10}$$

where $e^i = e^i(\theta, \phi)$ represents a unit three dimensional real vector parametrizing the two sphere. Its coordinate radius is given by $|z|/\sqrt{2}$. Inserting this ansatz into the action (2.1) we get the effective potential

$$\begin{aligned} V_{(c,d)}(z) = -\frac{S}{V} = \frac{4}{\pi g n (2\pi\alpha')^4} \Bigg\{ & |M|^2 |z|^4 + \frac{(2\pi\alpha')n}{3\sqrt{2}} \mathrm{Im}\left[3m\overline{M} z \bar{z}^2 + m'\overline{M} z^3\right] + \\ & + \frac{n^2(2\pi\alpha')^2}{8}\left(|m|^2 + \frac{|m'|^2}{3}\right)|z|^2 + \frac{n^2(2\pi\alpha')^2}{8}\mathrm{Re}[\mu^2 z^2]\Bigg\} \,. \end{aligned} \tag{2.11}$$

Since we are in a perturbed $AdS \times S^5$ geometry, the value of z which minimizes the previous potential should be interpreted as the location of the five brane along the radial direction of AdS, with the two sphere being an equator of the S^5.

Let us introduce the dimensionless complex parameters

$$b = \frac{m'}{m} \,, \qquad c = \frac{\mu^2}{m^2} \tag{2.12}$$

and also write

$$z = \frac{(2\pi\alpha')nm}{\sqrt{8}M} i x e^{i\varphi} \,. \tag{2.13}$$

Putting this into the five brane potential, we get

$$\begin{aligned} V_{(c,d)}(x,\varphi) = \left(\frac{m^4 n^2}{16\pi}\right)\frac{n}{g|M|^2} x^2 \Bigg\{ & x^2 - 2x\,\mathrm{Re}\left[e^{-i\varphi} + e^{3i\varphi}\frac{b}{3}\left(\frac{\overline{M}}{M}\right)^2\right] + \\ & + 1 + \frac{|b|^2}{3} - \mathrm{Re}\left[e^{2i\varphi} c \frac{\overline{M}}{M}\right]\Bigg\} \,. \end{aligned} \tag{2.14}$$

The critical points of the potential (2.11) (besides $z = 0$) are those real values of $\{x, \varphi\}$ that satisfy the equation

$$x^2 + Px + Q = 0 \,, \tag{2.15}$$

with the (generically complex) coefficients

$$P = -\frac{1}{2}\left(e^{-i\varphi} + 2e^{i\varphi} + e^{3i\varphi} b\left(\frac{\overline{M}}{M}\right)^2\right) \,, \tag{2.16}$$

$$Q = \frac{1}{2}\left(1 + \frac{|b|^2}{3} - e^{2i\varphi} c\left(\frac{\overline{M}}{M}\right)\right) \,. \tag{2.17}$$

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We could arrange several branes along different AdS radial directions. It means they have different D3 charge.[7] From the field theory point of view it corresponds to fuzzy (non-commutative) scalars in a reducible representation of SU(2), with different dimensions for several irreducible blocks. These configurations always leave some unbroken Abelian factors of the gauge group. They correspond to the Coulomb vacua.

Consider several five branes in the bulk, with D3 charge n_I for the (c_I, d_I) five brane. The effective potential is just the sum of all of them

$$V_{\text{total}} = \sum_I V_{(c_I,d_I)}(x_I, \varphi_I) \,. \tag{2.18}$$

Since the metric factors Z cancel in the PS approximation, we can take (2.18) as the actual full potential in the background created by the five branes. Due to the breaking of supersymmetry, we expect a unique vacuum for any given point in the parameter space (i.e. a unique five brane configuration is going to be selected). Furthermore, a numerical analysis shows that generically the vacuum energy is negative. As long as we confine ourselves to the classical PS approximation, the criteria is to look for the brane configuration with the most negative value of (2.18).

Observe that the potential (2.14) is proportional to the inverse of the small controlling parameter $g|M|^2/n$. This is good, because the selected brane configuration will correspond to the one where the PS approximation is the most optimal for each five brane involved. In fact, due to the constraint $N = \sum_I n_I$, the most favorable situations are the ones containing only one five brane. Hence, as long as we move in the region of parameter space corresponding to the PS approximation, the massive vacua are selected.

3. Analysis of the $\mathcal{N} = 0^*$ massive vacua

3.1 $\mathcal{N} = 1^*$ massive vacua and their dual description

Since the work of 't Hooft [31]–[33], we know which are the possible massive phases of four dimensional SU(N) gauge theories in which all fields transform trivially under the center of SU(N). They are classified by index N subgroups P, of the lattice $\mathbf{Z}_N \times \mathbf{Z}_N$, the group of external magnetic and electric charges by which this theory might be probed.[8]

Later on Donagi and Witten [34] found the integrable system behind the four dimensional $\mathcal{N} = 2$ SU(N) gauge theory with a massive hypermultiplet in the adjoint of the gauge group. They described the Coulomb branch moduli space of the theory in terms of N-fold covers of a genus N Riemann surface C, over a torus with modulus τ, the microscopic complexified gauge coupling of the underlying field theory. At generic

[7] The AdS location is proportional to the D3 charge, see (2.13).

[8] Charges are normalized such that $(0, 1)$ represents the smallest electric charge.

points in the Coulomb branch, the gauge group is broken to $\mathrm{U}(1)^{N-1}$. But at some particular points, r one-cycles of C degenerate, giving rise to r massless mutually local hypermultiplets. When the theory is perturbed to $\mathcal{N}=1$ by a mass term for the $\mathcal{N}=1$ chiral superfield in the $\mathcal{N}=2$ vector multiplet, the Coulomb branch flat directions are lifted and only the isolated points where the Riemann surface degenerates remain. These are the $\mathcal{N}=1^*$ vacua, where the $\mathcal{N}=1$ mass perturbation has produced the condensation of the r hypermultiplets associated to the singular one-cycles of C. Only at the points of maximal degeneration, where $r=N-1$, do we have a mass gap. At these points, C reduces to genus one, and the Donagi-Witten curve parametrizes an unramified N-fold cover of C over the Torus with modulus τ. These unramified N-fold covers are in one to one correspondence with index N subgroups P of the additive lattice $\mathbf{Z}_N \times \mathbf{Z}_N$.

If N is prime, there are only $1+N$ subgroups P. The one generated by $(0,1)$ represents the Higgs vacuum, where the smallest electrically charged particle condenses. In terms of the dual string theory description, it involves one $(0,1)$ five brane with D3 charge N. The remaining N subgroups are generated by $(1,s)$, $s=0,\ldots,N-1$, corresponding to confinement or oblique confinement. Each of these vacua has a dual description in terms of a single $(1,s)$ five brane.

If N is not prime, for every positive divisor $p>1$ of N and every r with $0\leq r\leq p-1$, there is an index N subgroup of $\mathbf{Z}_N \times \mathbf{Z}_N$ generated by $(0,p)$ and $(N/p,r)$. In terms of the dual type-IIB background, these massive vacua are represented either by p D5 branes, each with N/p D3 charge, or by N/p NS5 branes, each with p D3 charge. What determines the most effective description depends on the region of parameter space. For $N/gp^2 \gg 1$ the D5 brane description is the most favorable, and for $N/gp^2 \ll 1$ the NS5 brane takes over.

3.2 θ-angle dependence

We absorb the common phase of the fermion masses m and m' into $\theta = 2\pi\,\mathrm{Re}[\tau]$ such that we always take $m \in \mathbb{R}^+$.

Only five branes with NS charge are sensitive to the real part of τ.[9] For (q,r) five branes, where $q=N/p$ and $r=0,\ldots,p-1$, we can write $M=p\tilde{\tau}$, with $\tilde{\tau}=(q\tau+r)/p$. As was observed in [35], the Polchinski-Strassler solution only depends on this effective modular parameter. In particular, the five brane potential (2.14) becomes

$$V_{NS5}(x,\varphi) = \left(\frac{m^4N^2}{16\pi}\right)\mathrm{Im}\left[\frac{-1}{\tilde{\tau}}\right]x^2\Bigg\{x^2-2x\,\mathrm{Re}\left[e^{-i\varphi}+e^{3i\varphi}\frac{b}{3}\left(\frac{\tilde{\tau}^\star}{\tilde{\tau}}\right)^2\right]+ \\ +1+\frac{|b|^2}{3}-\mathrm{Re}\left[e^{2i\varphi}c\frac{\tilde{\tau}^\star}{\tilde{\tau}}\right]\Bigg\}. \qquad (3.1)$$

[9] This is in accordance with the picture of the NS5 brane and world-sheet instantons being the dual descriptions of the confinement phase and instanton effects, since these are θ-dependent dynamical effects in field theory.

As can be seen from the expressions for the potential and the location of its critical points, the precise analysis of the θ dependence can be highly intricate. But we can make some general qualitative observations. Since generically the value of the five brane potential at the minimum is negative, the most favorable would be the (q, r) five brane with the largest value of $\mathrm{Im}[-1/\tilde{\tau}]$. As we already observed, this is the same as the criteria for the validity of the classical supergravity description in the (q, r) five brane background, which to be concrete is

$$1 \ll \mathrm{Im}\left[\frac{-1}{\tilde{\tau}}\right] \ll p^2 . \tag{3.2}$$

Then, large p, and therefore "small" $q = N/p$, are preferred. Furthermore, given q, the smallest vacuum energy would be for that r for which $|\theta/2\pi + rp/N|$ is smallest.

3.3 Vacuum structure for $\theta = 0$

Since $\theta = 0$, we select $r = 0$ for the (q, r) five brane with $q \neq 0$. From (2.14), we see that for a $(q, 0)$ or a $(0, p)$ five brane, the vacuum energy is suppressed by a $1/q^2$ or $1/p^2$ factor, respectively. It is clear that $q = 1$ (one NS5 brane; complete confinement) or $p = 1$ (one D5 brane; complete Higgsing) are selected. Then, the vacuum structure is reduced to the analysis of these two (S-dual) cases.

But even then, the vacuum structure has a non-trivial dependence on the complex phases of the parameters b and c. R-symmetry selection rules give the R-charges: $R[z] = 2/3$, $R[b] = -8/3$ and $R[c] = -2$. This explains the combinations $e^{3i\varphi}b$ and $e^{2i\varphi}c$ appearing in (2.14), (2.16) and (2.17). If we want a five brane wrapped on a two sphere with non-zero size we should require a positive discriminant for the real part of equation (2.15):

$$\Delta = \mathrm{Re}[P]^2 - 4\,\mathrm{Re}[Q] \geq 0 . \tag{3.3}$$

This dependence is complicated. For fixed modulus $|b|$ and $|c|$, there may be phases of b and c which do not admit a positive discriminant. In particular, the overall sign of the last term in Q (see (2.17)) becomes crucial. At $\theta = 0$, this sign changes under S-duality. To see the consequences of this duality, we can simplify our analysis by taking b and c to be real.

For real parameters b and c, the phase φ vanishes. The equation for the critical points simplifies to the real solutions of

$$x^2 - \frac{1}{2}(3+b)x + \frac{1}{2}\left(1 + \frac{b^2}{3} - (-1)^{ns}c\right) = 0 , \tag{3.4}$$

where $ns = 0$ for the D5 brane and $ns = 1$ for the NS5 brane. Given a point in the space of parameters $\{N, g, b, c\}$, the vacuum will be described by the five brane with the most negative vacuum energy,

$$V_{ns}(x) = \left(\frac{m^4 N^2}{16\pi}\right) N g^{2ns-1} x^2 \left\{x^2 - 2x\left(1 + \frac{b}{3}\right) + 1 + \frac{|b|^2}{3} - (-1)^{ns}c\right\} . \tag{3.5}$$

Observe that the two potentials and their respective critical points are related by the interchange of $g \leftrightarrow 1/g$ and $c \leftrightarrow -c$. These correspond to mutually exclusive regions. The transformation involving the string coupling g is clearly a consequence of S-duality in $\mathcal{N} = 4$ SYM. The reason for the flip of sign in c can be understood by the following: for real parameters, the D5 brane world-volume corresponding to the two sphere is oriented in the $\{x^7, x^8, x^9\}$ subspace, as can be seen from (2.13); on the other hand, for the NS5 brane, it is spanned in the $\{x^4, x^5, x^6\}$ directions. Both orientations are related by a $\pi/2$ rotation of (2.13), which is equivalent as flipping the sign of c.

When $c = 0$, there is no distinction between the D5 and NS5 branes with respect to the location of the two sphere. The only difference rests on the global g dependence in the potential (3.5). When $g < 1$, the D5 vacuum is selected. For $g > 1$, the NS5 brane vacuum is the chosen one. This is in complete agreement with the underlying S-duality of the unbroken supersymmetric theory. A numerical analysis shows that only for $0 \leq b \leq 3$ there is a non-zero solution, $x_{\min} \neq 0$, of (3.4) where the value of the five brane potential evaluated at $x_{\min}$ is the absolute minimum.[10]

When $c > 0$, the region of the parameter b with a stable two sphere for the NS5 brane monotonically reduces as c is increased. In fact, for $c > 4/5$, there is no non-zero solution of (3.4) for the NS5 brane; it collapses to zero size, which lies outside of our approximation regime. On the other hand, for the D5 brane, increasing $c > 0$ increases the range of the parameter b. For $c \to \infty^+$, we have $|b_{max}| \to \frac{2}{5}\sqrt{30c}$.

When $c < 0$, the situation is completely reversed with respect to the D5 and NS5 branes. Negative c defines the region favorable for the NS5 brane.

The global picture is presented in figure 1 When $b = c = 0$, which is the $\mathcal{N} = 1^*$ supersymmetric point, the effective potential (3.5) looks like that of figure 2; the five brane sits at the minimum with non-zero x and the vacuum energy is zero. Close to the $\mathcal{N} = 1^*$ point, when b and c are smaller than one, what determines the vacuum is essentially the value of the string coupling, g. For $g < 1$ ($g > 1$) the D5 (NS5) brane vacuum is selected. Far from the origin, two symmetric branches appear: the Higgs branch for $c > 0$ and the confining branch for $c < 0$. In each of the branches, there is a critical line, defined by the vanishing of the smaller root of (3.4),

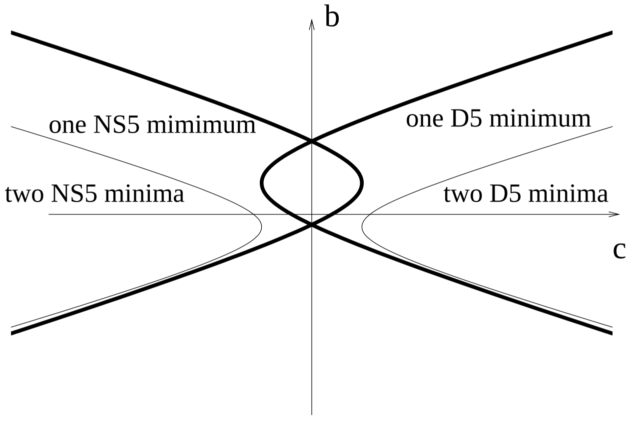

Figure 1: Phase structure within PS approximation.

[10]There is always a local minima at $x = 0$ with zero vacuum energy.

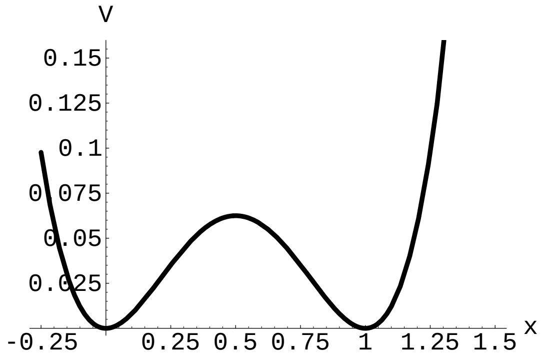

Figure 2: Supersymmetric potential.

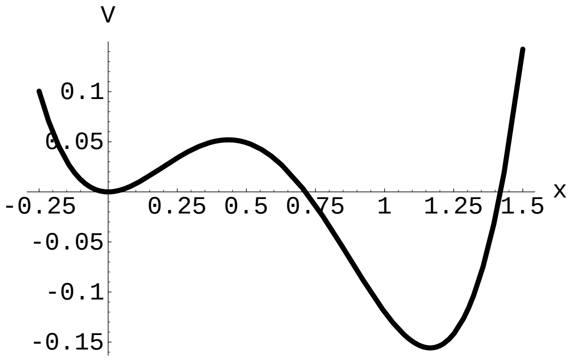

Figure 3: Phase with only one five brane minimum.

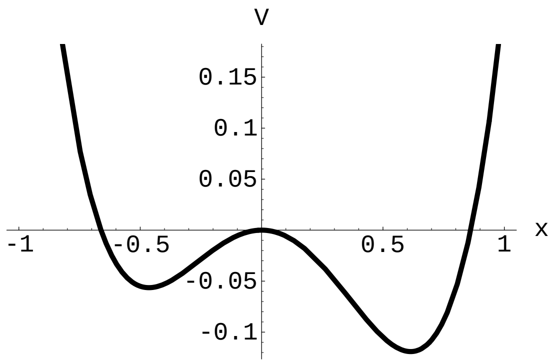

Figure 4: Phase with two five brane minima.

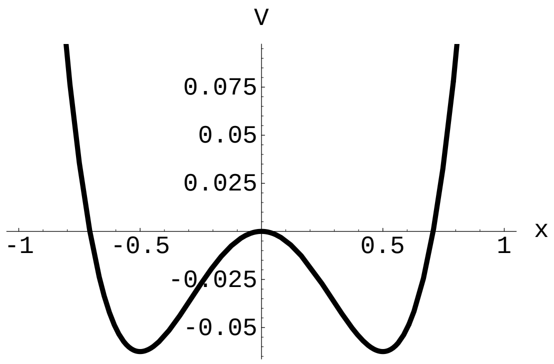

Figure 5: Potential at $b = -3$.

$x_-(b,c) = 0$. This line divides the branch into two regions. In one region, x_- is a maximum and the potential typically looks like in figure 3. In the other region, x_- is a local minimum, and the potential becomes like in figure 4. The minimum located at the largest value of the AdS radius, given by the bigger root x_+, is the absolute one. The exception is when $b = -3$, where both minima collide at the same absolute value of x and we just have a potential like that of figure 5.

Notice that we can scale the $\mathcal{N} = 1$ preserving masses to zero and keep non-zero m' and μ^2 supersymmetry breaking parameters. It corresponds to the asymptotic regions of the phase space shown in figure 1. Take the limit $\epsilon \to 0$, keeping fixed

$$\tilde{m} = \frac{m}{\epsilon}, \qquad \tilde{b} = b\epsilon, \qquad \tilde{c} = c\epsilon^2, \qquad \tilde{x} = x\epsilon. \tag{3.6}$$

The five brane location (2.13) is kept fixed and different from zero as far as $\tilde{b}^2 \leq (-1)^{ns}(24/5)\tilde{c}$. Finally, the effective potential (3.5) becomes

$$V_{ns}(\tilde{x}) = \left(\frac{\tilde{m}N^2}{16\pi}\right) N g^{2ns-1} \tilde{x}^2 \left\{ \tilde{x}^2 - \frac{2\tilde{b}}{3}\tilde{x} + \frac{\tilde{b}^2}{3} - (-1)^{ns}\tilde{c} \right\}. \tag{3.7}$$

4. Discussion

In this paper we have analyzed the SO(3)-invariant region of parameter space where the PS approximation is still valid. By this we mean the region where the relevant effective potential is given by expression (2.1), with its minimum given by stable five brane/D3 states wrapped on a round two-sphere of finite size. Beyond this region, either the potential (2.1) ceases to be a good approximation or the five brane collapses. Therefore, any significant extension of its limits would require qualitative (and probably quantitative) new work. In particular, it would be very interesting to reach the SU(3) invariant point $m = \mu^2 = 0$, $m' \neq 0$ and to find out if the five-dimensional singularity found in [22][11] is resolved and we have chiral symmetry breaking to SO(3).

Let us add some comments on the field theory physics. Most of the qualitative features exposed in [24] continue to hold here. For instance, the (p, q)-flux tubes were realized as (p, q) one-five/D3 near BPS bound states. Since our parameters m' and μ^2 break supersymmetry softly, we believe that these bound states are still there, at least in the PS approximation region. To compute their IR string tension, we need the near-shell background for the five brane metric and the dilaton. Qualitatively, they are given by expressions similar to those in [24], with the main difference being the location of the five brane along the AdS radial direction. The same can be said about the field theory spectrum, for instance the construction of baryons. Since only one vacuum is selected, the domain walls are lost. With respect to the condensates, there is a new dependence on the m' and μ^2 parameters. In particular, there is a new non-zero condensate related to the operator $\mathrm{tr}(F^2)$,[12] which can be obtained from the normalizable mode corresponding to the SU(3) singlet.

And finally, some words of caution are in order regarding the starting UV fixed point. If we want to restrict the analysis to classical supergravity, we have to take (at least) large N and strong bare 't Hooft coupling. Then, all the fields in the $\mathcal{N} = 4$ supermultiplet continue to be relevant at the RG-invariant generated scale. As noticed in [36], there are qualitative new features in the $\mathcal{N} = 1^\star$ vacua with respect the $\mathcal{N} = 1$ SYM vacua. Decoupling of the additional $\mathcal{N} = 4$ fields involves decreasing the 't Hooft coupling, whose inverse parameter controls the α' corrections in the AdS/CFT correspondence.

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[11] Which satisfies the criteria established in [23].

[12] We remind that without significant supersymmetry constraints, there is mixing between composite operators. See [35] for a discussion in the context of the $\mathcal{N} = 1^\star$ theory.

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